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A New Solution Approach: To find optimal solution to the Fully Fuzzy time cost trade-off problem

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Abstract

In this paper we focus on the solution procedure of the Fuzzy crashing time problem with Fuzzy parameters. We propose a new method for the solution of Fuzzy Trade off problems without converting them into classical Fuzzy problems. Numerical examples are provided to illustrate the solution procedure developed in this paper and it is easy to understand and to apply for finding Fuzzy optimal solution of Fuzzy Trade off problems in real life situation.

Key words Fuzzy sets, Fuzzy number, Triangular Fuzzy number, Fuzzy arithmetic, Fuzzy Ranking.

1 Introduction

The main aspects of project scheduling are Time Cost Trade Off problem. It may be defined as a process to identify optimal construction activities for speeding up, and for deciding by how much so as to attain the optimal time and cost. Since there is a hidden tradeoff relationship between project time and cost, it might be difficult to predict whether the total cost (i.e., the direct and indirect costs) would increase or decrease as a result of the schedule compression. In real-life situations, decision-makers should always consider the trade-off between project completion time and project cost. In 1961, Kelly [12] first did research on the TCTP, which is one branch of the project scheduling problem. In the next 50 years, the research on the TCTP mainly focused on the deterministic cases Mobinia et al.[17], Phillips et al.[18], Weglarz et al.[27].

For solving the deterministic TCTP, the common analytical methods were linear programming and dynamic programming Talbot et al.[23], Heberta et al.[9]. Besides, some heuristic algorithms, such as genetic algorithm Azaron et al.[1], Kim et al.[13] were also applied. All projects have risks and uncertainties. Fuzzy set theory is more appropriate to model these problems. Wang et al.[26] developed a model to project scheduling with fuzzy information. Leu et al.[14] developed a fuzzy optimal model to formulate effects of both certain activity duration and resource constraint. In general, most of the existing methods provide only crisp solutions for the fully fuzzy trade-off problem. In this paper we propose a simple method, for the solution of fully fuzzy time cost trade-off problem without converting them in to crisp fully fuzzy linear programming problem. The rest of this paper is organized as: In section 2, we recall 2 the basic concepts and the results of triangular fuzzy numbers and their arithmetic operations. In section 3, we define fully fuzzy time cost trade-off problem and prove some of the important theorems, related results. In section 4, numerical examples are provided to illustrate the methods proposed in this paper for the fully fuzzy time cost trade-off problem without converting them to classical time cost trade-off problem and the results obtained are discussed.

2 Preliminaries

The aim of this section is to present some notations, notions and results which are of useful in our further consideration.

2.1 Fuzzy number

A fuzzy set Ã defined on the set of feal numbers R is said to be a fuzzy number if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ has the following characteristics

(i) Ã is normal. It means that there exists an $x \in R$ such that $\mu_{\tilde{A}}(x) = 1$

(ii) $\tilde{A}$ is convex.

It means that for every $x_1, x_2 \in R$,

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \lambda \in [0,1]$$

(iii) $\mu_{\tilde{A}}$ is upper semi continous.

(iv)supp ($\tilde{A}$) is bounded in R.

Definition 2.2. A trapezoidal fuzzy number Ã is denoted as $\tilde{A} = (a_1, a_2, a_3)$ and is defined by the membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & x = a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

We denote this triangular fuzzy number by $\tilde{A} = (a_1, a_2, a_3)$ We use F(R) to denote the set of all triangular fuzzy numbers. Also if $m = a_2$ represents the modal value or midpoint, $\alpha = a_2 - a_1$ represents the left spread and $\beta = a_3 - a_2$ represents the right spread of the triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ then the triangular fuzzy number can be represented by the triplet $\tilde{A} = (m, \alpha, \beta)$ That is $\tilde{A} = (a_1, a_2, a_3) = (m, \alpha, \beta)$

2.3 Definition

Any triangular number $\tilde{A} = (a_1, a_2, a_3)$ can be written as $\tilde{A} = (m, \alpha, \beta)$ and it is said to be symmetric if $\alpha = \beta$.

2.4 Ranking of triangular fuzzy numbers

Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function based on their graded means. That is, for every $\Re : F(R) \to R$ which maps each triangular fuzzy number into a real number, where a natural order exists

For every triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$, ranking function $\Re : F(R) \to R$ is defined by graded mean as

$$\Re(\tilde{A}) = \frac{(a_1 + 4a_2 + a_3)}{6}$$

For any two fuzzy triangular numbers $\tilde{A}=(a_1,a_2,a_3)$ and $\tilde{B}=(b_1,b_2,b_3)$ we have the following comparison

$$(i)\ \tilde{A} \succ \tilde{B} \text{ if and only if } \Re(\tilde{A}) \succ \Re(\tilde{B})$$

$$(ii)\ \tilde{A} \prec \tilde{B} \text{ if and only if } \Re(\tilde{A}) \prec \Re(\tilde{B})$$

$$(iii)\ \tilde{A} \approx \tilde{B} \text{ if and only if } \Re(\tilde{A}) \approx \Re(\tilde{B})$$

$$(iv)\ \tilde{A} - \tilde{B} \approx \tilde{0} \text{ if and only if } \Re(\tilde{A}) - \Re(\tilde{B}) = 0$$

A triangular fuzzy number $\tilde{A}=(a_1,a_2,a_3)\in F(R)$ is said to be positive if $\Re(\tilde{A})\succ 0$

Also $\tilde{A}\approx\tilde{0}\ if\ \Re(\tilde{A})=0$ and $\tilde{A}\prec\tilde{0}$ if $\Re(\tilde{A})\prec 0$

If $\Re(\tilde{A})=\Re(\tilde{B})$ then the triangular numbers $\tilde{A}$ and $\tilde{B}$ are said to be equivalent and it is denoted by $\tilde{A}\approx\tilde{B}$.

2.5 Arithmetic operations on triangular fuzzy number

A new fuzzy arithmetic introduced by Ming Ma et. al.[16] has been used in this dissertation as it seems to be relevant for modeling, analysis and applications in engineering. This new fuzzy arithmetic is based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice *L*. That is for *a; b* ∈ *L*, we define $a \vee b$ = max *{a; b}* and $a \wedge b$ = min *{a; b}*.

For any two arbitrary triangular fuzzy numbers $\tilde{A}=(a_1,a_2,a_3)$ and $\tilde{B}=(b_1,b_2,b_3)$ and $*=(+,-,\times,\div)$; the arithmetic operations on the triangular fuzzy numbers are defined by $\tilde{A}*\tilde{B}=(m_1*m_2,\ \alpha_1\vee\alpha_2,\beta_1\vee\beta_2)$: In particular for any two triangular fuzzy numbers $\tilde{A}=(m_1,\alpha_1,\beta_1)$ and $\tilde{B}=(m_2,\alpha_2,\beta_2)$ the arithmetic operations are defined by,

(i) *Addition*

$$\tilde{A}+\tilde{B}=(m_1+m_2,\ \max\{\alpha_1,\ \alpha_2\},\max\{\beta_1,\ \beta_2\})$$

(ii) *Subtraction*

$$\tilde{A}-\tilde{B}=(m_1-m_2,\ \max\{\alpha_1,\ \alpha_2\},\max\{\beta_1,\ \beta_2\})$$

(ii) *Multiplication*

$$\tilde{A}\times\tilde{B}=(m_1\times m_2,\ \max\{\alpha_1,\ \alpha_2\},\max\{\beta_1,\ \beta_2\})$$

(ii) *Division*

$$\tilde{A}\div\tilde{B}=(m_1\div m_2,\ \max\{\alpha_1,\ \alpha_2\},\max\{\beta_1,\ \beta_2\})$$

3 Main Results

3.1 Mathematical formulation

Consider a project model G = (N,A) which is a directed and connected network, where N is the set of n nodes and A is the set (*i; j*) ∈ *A arcs* $\tilde{T}_{ij}$ denote the fuzzy activity time of(*i; j*) ∈ *A*.

$$\text{Max } \tilde{D}_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{T}_{ij}\tilde{x}_{ij}$$

Subject to the constraints,

$$\sum_{j=1}^{n} \tilde{x}_{1j} = \tilde{1}$$

$$\sum_{j=1}^{n} \tilde{x}_{ij} = \sum_{k=1}^{n} \tilde{x}_{ki} = \tilde{1},\ i = 1,2,3...n-1$$

$$\sum_{k=1}^{n} \tilde{x}_{kn} = \tilde{1}\ ;\ \tilde{x}_{ij} \geq \tilde{0}\ ;(i, j) \in A$$

Where $\tilde{x}_{ij}$ is the fuzzy decision variable denoting the amount of flow in (*i; j*) ∈ *A* and the constraints represents the conservatives of flow at each node, indicating that the flow may be neither created nor destroyed in the fuzzy project network.

3.2 PROPOSED ALGORITHM

1. Draw the network.
2. Determine the fuzzy critical path and the fuzzy normal durations.Also identify the fuzzy critical activities.
3. Find the total fuzzy normal cost and the fuzzy normal durations of the project. 4. Compute the fuzzy cost slope for each activity.
Fuzzy cost slope ≈ (Fuzzy crash cost- Fuzzy Normal cost)÷
(Fuzzy Normal Duration - Fuzzy crash Duration)

5. Crash the fuzzy critical activity of the least cost slope first to the maximum extent possible so that project duration is really reduced.
6. Calculate the new fuzzy direct cost by cumulatively adding the cost of the crashing to the current direct cost.
7. When fuzzy critical activities are crashed and the fuzzy duration is reduced other paths may also become critical.When more than one fuzzy critical paths exists in the fuzzy project network, fuzzy project duration can be reduced only when either the duration of a critical activity common to all fuzzy critical paths is reduced or different suitable activities on different fuzzy critical paths are simultaneously reduced.
8. Stop when the total fuzzy cost is minimum. This gives optimum schedule.

4 Numerical Examples

The following numerical example is taken from the paper ”A Direct approach to fuzzy time cost trade off” by M.Evangelin Jebaseeli and D.Paul Dhayabaran [7].

Example 4.1

Activity	Crash time	Normal time	Normal cost	Crash cost
1-2	(22,21,22)	(23,24,25)	(400,500,600)	(700,800,900)
2-3	(18,18,18)	(20,20,20)	(500,500,500)	(600,600,600)
2-4	(19,20,21)	(22,22,22)	(600,700,800)	(900,900,900)
3-4	(16,16,16)	(20,20,20)	(600,600,600)	(800,800,800)
4-5	(18,19,20)	(21,22,23)	(400,550,700)	(800,850,900)
4-6	(22,22,22)	(23,23,23)	(700,700,700)	(800,800,800)
5-6	(18,18,18)	(19,19,19)	(500,600,700)	(800,800,800)
6-7	(15,16,17)	(18,18,18)	(400,400,400)	(900,900,900)

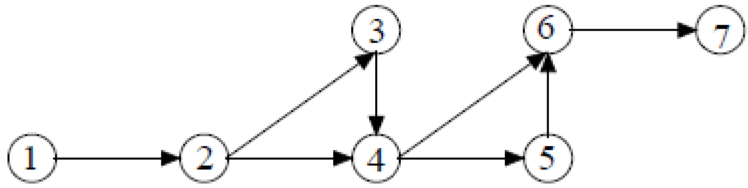

Figure 1: Fuzzy project network

let us express all the triangular fuzzy numbers based upon both location index and fuzziness index functions. That is in the form of $\tilde{A} = (a_1, a_2, a_3) = (m, \alpha, \beta)$.

Table 1: Initial Table

Activity	Crash time	Normal time	Normal cost	Crash cost	Slope
1-2	(21,1-r,1-r)	(24,1-r,1-r)	(500,100-100r,100-100r)	(800,100-100r,100-100r)	(500,100-100r,100-100r)
2-3	(18,0,0)	(20,0,0)	(500,0,0)	(600,0,0)	(50,0,0)
2-4	(20,1-r,1-r)	(22,0,0)	(700,100-100r,100-100r)	(900,0,0)	(100,100-100r,100-100r)
3-4	(16,0,0)	(20,0,0)	(600,0,0)	(800,0,0)	(50,0,0)
4-5	(19,1-r,1-r)	(21,1-r,1-r)	(550,150-150r,150-150r)	(850, 50-50r,50-50r)	(100,150-150r,150-150r)
4-6	(22,0,0)	(23,0,0)	(700,0,0)	(800,0,0)	(100,0,0)
5-6	(18,0,0)	(19,0,0)	(600,100-100r,100-100r)	(800,0,0)	(200,100-100r,100-100r)
6-7	(16,1-r,1-r)	(18,0,0)	(400,0,0)	(900,0,0)	(250,1-r,1-r)

Total fuzzy duration ≈ (123, 1-*r*, 1-*r*) days
Total fuzzy cost ≈ Indirect cost + Direct cost
≈ (123,1-*r*,1-*r*) (100,0,0) + (4550,150-150*r*,150-150*r*)
≈ (16850,150-150*r*,150-150*r*)

Table 2: Solution table

Stage	Crash	Current Duration	Direct cost	Indirect cost	Total cost
0	0	(123,1-r,1-r)	(4550,150-150r,150-150r)	(12300,1-r,1-r)	(16850,150-150r,150-150r)
1	2-3	(121,1-r,1-r)	(4650,150-150r,150-150r)	(12100,1-r,1-r)	(16750,150-150r,150-150r)
2	3-4	(117,1-r,1-r)	(4850,150-150r,150-150r)	(11700,1-r,1-r)	(16550,150-150r,150-150r)
3	1-2	(114,1-r,1-r)	(5150,150-150r,150-150r)	(11400,1-r,1-r)	(16550,150-150r,150-150r)
4	4-5	(111,1-r,1-r)	(5450,150-150r,150-150r)	(11100,1-r,1-r)	(16550,150-150r,150-150r)
5	5-6	(110,1-r,1-r)	(5650,150-150r,150-150r)	(11000,1-r,1-r)	(16550,150-150r,150-150r)

Optimal fuzzy duration is (111,1-*r*,1-*r*) time units.
Optimal fuzzy cost ≈ (16550,150-150*r*,150-150*r*) cost units.
But by the existing method, optimal fuzzy duration is (104,110,116) time units.
Optimal fuzzy cost ≈ (16800,17650,18500) cost units.

Example 4.2.

The following numerical example is taken from the paper ”A Comparative study on Fully Fuzzy time cost trade off” by M.Evangelin Jebaseeli and D.Paul Dhayabaran [6].

Activity	Crash time	Normal time	Normal cost	Crash cost
1-2	(22,24,26)	(28,28,28)	(600,600,600)	(800,800,800)
1-3	(21,21,21)	(23,24,25)	(700,700,700)	(800,850,900)
2-3	(22,22,22)	(23,23,23)	(500,600,700)	(800,800,800)
2-5	(18,20,22)	(23,24,25)	(600,600,600)	(900,900,900)
3-4	(15,15,15)	(16,17,18)	(700,800,900)	(900,1000,1100)
4-6	(15,16,17)	(18,19,20)	(700,800,900)	(1100, 1100,1100)
5-6	(15,15,15)	(18,18,18)	(800,800,800)	(1000,1000,1000)
5-7	(23,25,27)	(28,28,28)	(600,700,800)	(800,900,1000)
6-8	(19,19,19)	(20,20,20)	(400,400,400)	(400,500,600)
7-8	(14,16,18)	(20,20,20)	(400,500,600)	(800,900,1000)
7-9	(20,21,22)	(23,24,25)	(600,600,600)	(700,750,800)
8-9	(20,20,20)	(22,22,22)	(500,500,500)	(800,800,800)

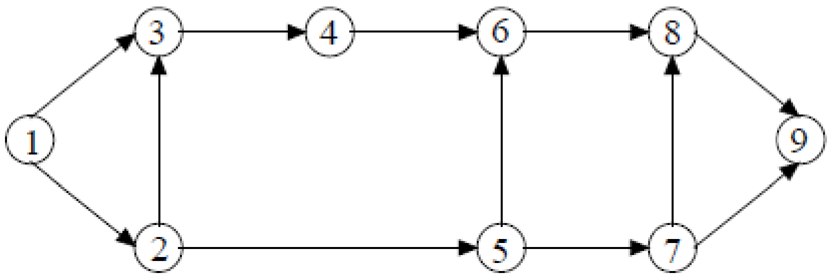

Figure 2: Fuzzy project network

Table 3: Initial table

Activity	Crash time	Normal time	Normal cost	Crash cost	Slope
1-2	(24,2-2r,2-2r)	(28,0,0)	(600,0,0)	(800,0,0)	(50,2-2r,2-2r)
1-3	(21,0,0)	(24,1-r,1-r)	(700,0,0)	(850,50-50r,50-50r)	(50,50-50r,50-50r)
2-3	(22,0,0)	(23,0,0)	(600,100-100r,100-100r)	(800,0,0)	(200,100-100r,100-100r)
2-5	(20,2-2r,2-2r)	(24, 1-r,1-r)	(600,0,0)	(900,0,0)	(75,2-2r,2-2r)
3-4	(15,0,0)	(17,1-r,1-r)	(800,100-100r,100-100r)	(1000,100-100r,100-100r)	(100,100-100r,100-100r)
4-6	(16,1-r,1-r)	(19, 1-r,1-r)	(800,100-100r,100-100r)	(1100,0,0)	(100,100-100r,100-100r)
5-6	(15,0,0)	(18,0,0)	(800,0,0)	(1000,0,0)	(66.7,0,0)
5-7	(25,2-2r,2-2r)	(28,0,0)	(700, 100-100r,100-100r)	(900,100-100r,100-100r)	(66.7,100-100r,100-100r)
6-8	(19,0,0)	(20,0,0)	(400,0,0)	(500,100-100r,100-100r)	(100,100-100r,100-100r)
7-8	(16,2-2r,2-2r)	(20,0,0)	(500, 100-100r,100-100r)	(900,100-100r,100-100r)	(100,100-100r,100-100r)
7-9	(21,1-r,1-r)	(24, 1-r,1-r)	(600,0,0)	(750,50-50r,50-50r)	(50,50-50r,50-50r)
8-9	(20,0,0)	(22,0,0)	(500,0,0)	(800,0,0)	(150,0,0)

Total fuzzy duration ≈ (129,1-*r*,1-*r*) time units
Total fuzzy cost ≈ (129,1-*r*,1-*r*)(150,0,0) + (7600,100-100*r*,100-100*r*)
≈ (26950,100-100*r*,100-100*r*) cost units.

Table 4: Solution table

Stage	Crash	Current Duration	Direct cost	Indirect cost	Total cost
0	0	(129,1-r,1-r)	(7600,100-100r,100-100r)	(19350,1-r,1-r)	(26950,100-100r,100-100r)
1	1-2	(125,2-2r,2-2r)	(7800,100-100r,100-100r)	(18750,2-2r,2-2r)	(26550,100-100r,100-100r)
2	3-4	(123,2-2r,2-2r)	(8000,100-100r,100-100r)	(18450,2-2r,2-2r)	(26450,100-100r,100-100r)
3	4-6	(120,2-2r,2-2r)	(8300,100-100r,100-100r)	(18000,2-2r,2-2r)	(26300,100-100r,100-100r)
4	6-8	(119,2-2r,2-2r)	(8400,100-100r,100-100r)	(17850,2-2r,2-2r)	(26250,100-100r,100-100r)
5	8-9	(117,2-2r,2-2r)	(8700,100-100r,100-100r)	(17550,2-2r,2-2r)	(26250,100-100r,100-100r)
6	2-3	(116,2-2r,2-2r)	(8900,100-100r,100-100r)	(17400,2-2r,2-2r)	(26300,100-100r,100-100r)

Fuzzy optimal duration is (117,2-2*r*,2-2*r*) time units.
Fuzzy optimal cost (26250,100-100*r*,100-100*r*) cost units.
But by the existing method, optimal fuzzy duration is (109,116,123) time units.
Optimal fuzzy cost is ≈ (26250,27900,29550) cost units.

∴ The result obtained by the proposed method is better than existing method .

5 Conclusion

In this chapter, a fuzzy version of fully fuzzy time cost trade off problems without converting them into a classical one is developed. Numerical example discussed by M.Evangelin Jebaseeli and D.Paul Dhayabaran [[7], [6]]are solved using the proposed method without converting the given problem to crisp equivalent problem and a comparison study has been made. The fuzzy optimal cost as well as fuzzy optimal durations obtained by the proposed algorithm is more viable than the existing methods, since it has less spread. Solutions with various satisfactory levels obtained by the proposed method helps the decision maker to take suitable decision according to the situation.

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